A Monetary Theory of Entrepreneurship and Differential Effects of Monetary Policy*

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Abstract

This paper studies a pure currency economy where agents can choose two occupations: workers and entrepreneurs. Money is valuable even though borrowing is allowed because it solves an asynchronization problem of expenditure and revenue. We show that steady-state inflation always raises entrepreneurship. Moreover, inflation always reduces output if the business cost (BC) (i.e., the fixed cost of entrepreneurship) is low. However, it first raises and then reduces output if the BC is high, and the threshold of this nonmonotonicity is increasing in the BC. The evidence is consistent with the positive effects of long-run inflation on entrepreneurship.

JEL Classification: E31, E52, J24, L26.

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1 Introduction

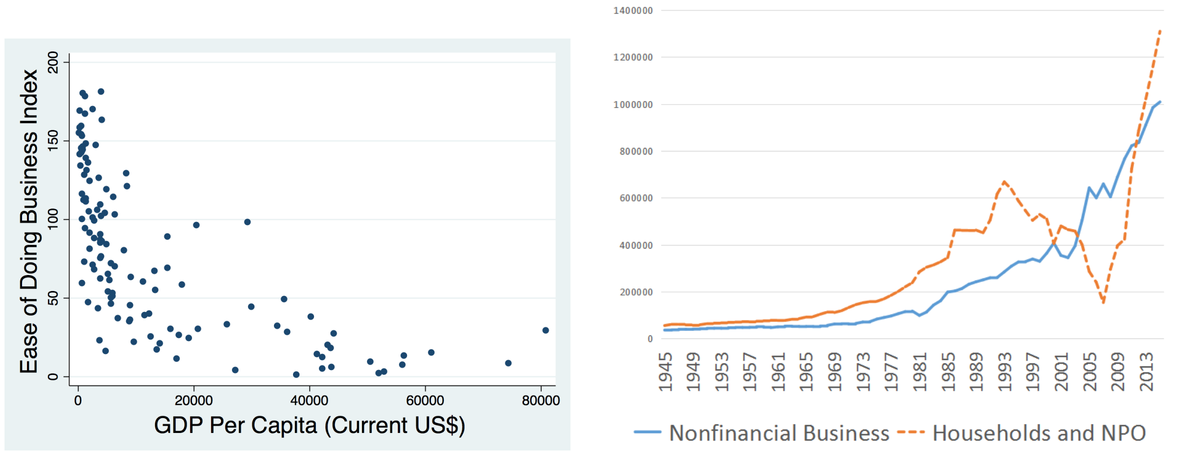
Economists have been building general equilibrium models to study the effects of inflation ever since the work of Tobin (1965) and Sidrauski (1967). This literature has focused on two questions: are inflation and growth/output related in the long run? If so, how? These questions are important because, in the fiat money era, every central bank faces the problem of choosing the growth rate of the money supply and thus the long-run inflation rate. They also concern the validity of the assumed vertical long-run Phillips curve in many short-run monetary models. The payoff to understanding these questions could be vast. Over the years, the literature has examined various channels through which steady-state inflation can affect output. Many theories predict a negative relationship. Some argue that the relationship could be nonmonotonic. Still others doubt that any relationship exists at all. The evidence is mixed, and disagreement and theoretical debate continue.[1] However, if these different monetary theorists can agree on one thing, it is the rarely questioned conventional wisdom in monetary theory that different countries must share similar trade-offs or unrelatedness between long-term inflation and output.

This paper proposes a theory of how the relationship between inflation and output can be systematically and qualitatively different across countries. The theory is based on a novel channel: the occupational choices of becoming an entrepreneur. In our model economy, agents can freely choose between two occupations: workers and entrepreneurs. Money flows back and forth between them: workers use cash to purchase goods from entrepreneurs, and the latter use it to pay wages to the former. If a period ends right after (before) workers get paid, then it appears that workers (entrepreneurs) hold money across periods and thus are burdened with the inflation tax. We show that the timing assumptions do not matter in our setting: steady-state inflation always increases entrepreneurship. Moreover, inflation always reduces output if the business cost (BC) (i.e., the fixed cost of entrepreneurship) is low. However, it first raises and then reduces output if the BC is high, and the threshold of this nonmonotonicity is increasing in the BC.

Of course, this is not the first theory of how long-run inflation can have nonmonotonic effects on output. Previous examples include Azariadis and Smith (1996), Shi (1997), and Lagos and Rocheteau (2005).[2] What is novel about the theory in this paper

[1] These different theoretical channels include capital accumulation, labor supply, cash goods—credit goods substitution, search frictions, and the trade of goods that promote growth. See Stockman (1980), Lucas and Stokey (1987), Cooley and Hansen (1989, 1991), Lagos and Wright (2005), Aruoba et al. (2007), and Berentsen et al. (2009) for a sample of theoretical works and Gillman and Kejak (2005) for a survey. See Bullard and Keating (1995), Barro (1996), Atkeson et al. (2004), and Lopez-Villavicencio and Mignon (2011) for a sample of empirical studies and Bullard (1999) for a survey.

[2] The nonmonotonic effects exist in all countries in Azariadis and Smith (1996) and depend on parameter values in the other two papers, which focus on the theoretical possibilities. We do not have par-



(a) Income Level and Ease of Doing Business (lower means better)

(b) Checkable Deposits and Currency in the US

Figure 1: Business Costs and Business Liquidity

Notes: In Panel (a), the Ease of Doing Business Index is constructed by the World Bank and ranks economies from 1 to 190, with first place being the best. A high ranking (a low numerical index) means that the regulatory environment is conducive to business operation. In Panel (b), the data are from the Financial Accounts of the United States (Z.1) from the Board of Governors of the Federal Reserve System.

is that the relationship between inflation and output can be systematically and qualitatively different across countries: the nonmonotonic effects exist in high-business-cost (HBC) countries but not in low-business-cost (LBC) countries. The BC hereafter means the fixed cost of entrepreneurship, and it does vary across countries. If we only look at the official fees and taxes for business registration paid by a typical start-up firm in 2004, they make up 534% of GDP per capita in Cambodia but only 1% in the United Kingdom, according to the World Bank. More generally, the BC also includes the cost owing to weak law enforcement, corruption, poor infrastructure, and so on, which are also very different across countries. Panel (a) of Figure 1 plots an Ease of Doing Business Index (the lower the number, the better) against the income level. It suggests that the HBC countries are more likely to be developing countries. Thus, the theory here calls for caution when applying the monetary models that enjoy success in the more advanced economies to the developing world.

Furthermore, the theory also predicts that the threshold of nonmonotonicity is increasing in the BC and thus different across countries. This is consistent with the observation that many emerging market economies have a greater tolerance for inflation. But it also suggests that detecting the nonmonotonic effect empirically can be more challenging than it appears. Because previous empirical studies have ignored the BC, this may help us to understand the existing mixed empirical evidence on the effects of

ticular reason or evidence to believe those parameter values should be systematically different across countries.

long-run inflation on output/growth.

We also confront the main theoretical prediction, that long-run inflation encourages the stock of entrepreneurship, with the data. An instrumental variable approach allows us to identify the causal effects. Building on the political economy theory of Bullard et al. (2012), we make use of the exogenous variations of inflation caused by age structure. We first confirm the relationship between inflation and age structure and then use it to confirm the positive effects of long-run inflation on entrepreneurship. To show excludibility, we use the fact that the eurozone is a monetary union. The age structure of an individual country would not affect the monetary policy of the union, so the inflation channel is not effective. If there are other channels through which age structure affects entrepreneurship, they should be still effective in the monetary union. We find that age structure is associated with entrepreneurship outside the monetary union but not inside the union, indicating that there is no evidence for these other channels.

In terms of monetary theory, we extend traditional pure-currency-economy models (e.g. Lucas 1980) in three aspects. First, firms are now owned and operated by entrepreneurs and agents can freely switch occupations. Second, money is needed for the transaction of both consumption goods and labor input, whereas existing monetary theories focus on the demand for liquidity by either consumers or producers (see, e.g., Meltzer 1963, Blinder 1987, Christiano and Eichenbaum 1992). This is important because in reality both households and firms hold currency and checkable deposits, as shown in Panel (b) of Figure 1. Relatedly, a large literature in finance studies the demand for money by firms (see, e.g., Mulligan 1997, Bover and Watson 2005, and Bates et al. 2009). Our setting helps to keep track of the flow of money in the economy and to clarify the difference between requiring workers or entrepreneurs to hold money across periods: it is about how we define a period in time. Perhaps surprisingly, the timing assumption does not matter in our setting. Lastly, the use of money in exchange is not guaranteed by the imposition of the usual cash-in-advance constraint, which requires that purchases of goods must necessarily be paid for by currency held over from the preceding period. In our model, agents have access to financial markets so that they can borrow the liquidity needed.[3] What is important for the money to circulate is that agents consume gradually during a period, and yet wage payments happen once in a period. The economy thus has a natural asynchronization of expenditure and revenue.

Another message of the paper is that entrepreneurship not only is a crucial factor in our understanding of other areas of macroeconomics, but also is important

[3]This is related to a point made by Kohn (1981): since borrowing and lending net to zero, the money borrowed by one person to evade the "cash constraint" represents a corresponding tightening of the constraint of the lender.

for the understanding of monetary policies. Little effort has been made to examine the effects of inflation on and through entrepreneurship. Few exceptions include Silviera and Wright (2010) and Chiu et al. (2017). These studies focus on the intensive margin (how much each entrepreneur produces) and leave alone the extensive margin, entrepreneurial occupational choices, which is one of the central pieces in the entrepreneurship literature (see a survey in Quadrini 2009).[4] Many studies in that literature have emphasized the role of borrowing constraints or risk aversion (see Evans and Jovanovic 1989, Hurst and Lusardi 2004, Cagetti and De Nardi 2004, Buera and Yongseok Shin 2013, Vereshchagina and Hopenhayn 2009, and the survey in Simon 2009). This paper is the first to model entrepreneurial occupational choices using a monetary framework and complements the entrepreneurship literature by pointing to an alternative factor that has been overlooked: long-run inflation. As mentioned above, we also provide empirical evidence of this channel.

The rest of the paper is organized as follows. Section 2 introduces the model with both types of timing assumptions and discusses the normative implications of the theory. Section 3 confronts our main prediction with the data. Section 4 discuss the empirical results, and Section 5 concludes.

2 The Model

This section has three parts. The first part describes the physical environment and the planner's problem. The second part discuss the timing assumption. The third part introduces the monetary market economy when a period ends right before workers get paid. The fourth part discusses the alternative assumption that a period ends right after workers get paid.

2.1 Environment and the Planner's Problem

There is measure one of identical agents. A fraction n of them are entrepreneurs, and the remaining $1-n$ fraction are workers. Each agent discounts the future at rate β with periodic utility: $U^i(x_t, y_t) = u(x_t) + y_t - v_i$, where x, y, v_i are the market good, the endowment good, and periodic occupational costs, respectively. The variable i could be e or w for entrepreneurs or workers. In every period, each worker supplies one unit of labor, and each entrepreneur uses labor to produce the market good with production function $f(\ell_t)$, where ℓ_t is labor input. Assume that $u(x)$ and $f(\ell)$ are increasing and concave, with $u'(0) = \infty$ and $f'(0) = \infty$. Every agent receives the same endowment good Y in any period. It is worth mentioning that the quasi-linear utility

[4]Relatedly, Bergin and Corsetti (2008) consider how short-run monetary policy shocks affect firm entry.

and a large endowment of Y make the model tractable because when we later allow agents to switch occupations, they finish the transition within a period.[5]

Before studying endogenous entrepreneurship in a monetary market economy, we will look at the planner's problem so as to understand the efficiency properties of the model. Now suppose that a social planner can freely allocate agents to the two types of occupations. The trade-offs are twofold: first, more entrepreneurs means more production units, but total labor input as well as labor input per production unit would decrease. Second, since v_w and v_e are generally different, entrepreneurship affects the total periodic fixed costs paid by society as a whole.

We assume the planner puts equal weight on each agent, and she chooses occupations (with implied fixed costs from work), as well as x and y for everyone. The planner faces three resource constraints: (a) total labor input is equal to the number of workers, (b) total consumption of the market good is equal to production, and (c) total y adds up to Y. Since the function $u(x)$ is the same for everyone and is concave, the planner would choose the same x for everyone. This does not necessarily mean that the planner equalizes every agent's utility level,[6] but she would always choose n to solve the following problem in steady states:

$$\max_n u[nf(\frac{1-n}{n})] - nv_e - (1-n)v_w + Y. \tag{1}$$

Because of the Inada condition, it is never optimal to set n to be 0 or 1. We can show that the second-order condition is satisfied and the objective function of the planner is concave. So there is a unique maximizer of the problem. Specifically, the first-order condition is as follows:

$$u_x[nf(\frac{1-n}{n})][f(\frac{1-n}{n}) - \frac{1}{n}f_\ell(\frac{1-n}{n})] = v_e - v_w. \tag{2}$$

The left-hand side of (2) is the marginal changes in utility from goods consumption arising from changes in entrepreneurship, whereas the right-hand side is the marginal fixed cost differential. Let n_{op} be the socially optimal entrepreneurship level that satisfies the above condition and n_{mx} be the measure of entrepreneurs that maximizes output. Obviously, n_{mx} also maximizes social welfare if $v_w - v_e$ equals zero. In the special case in which $v_w = v_e$, the objective is just to maximize output, which is concave in n, as shown in Figure 1. For the general case, Figure 2 shows two graphs of the objective function when $v_w - v_e$ is negative and positive, respectively. When $v_w - v_e > 0$, the fixed costs of being a worker are greater, and we say entrepreneurship is preferable to

[5]Another potential benefit, which is not pursued in this paper, is that if we introduce idiosyncratic (productivity) shocks, the distribution of state variables would be degenerate, similar to that in Lagos and Wright (2005).

[6]The utility function has a linear term in y, so giving one agent a little more of y at the cost of another agent does not go against the requirement for equal weights. In any event, when the planner adds up everyone's utility, these measures of y add up to Y.

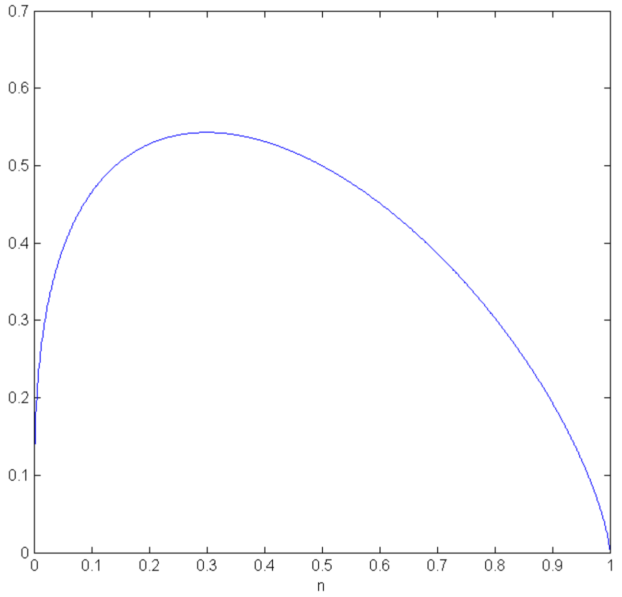


Figure 2: Output as a Function of Entrepreneurship

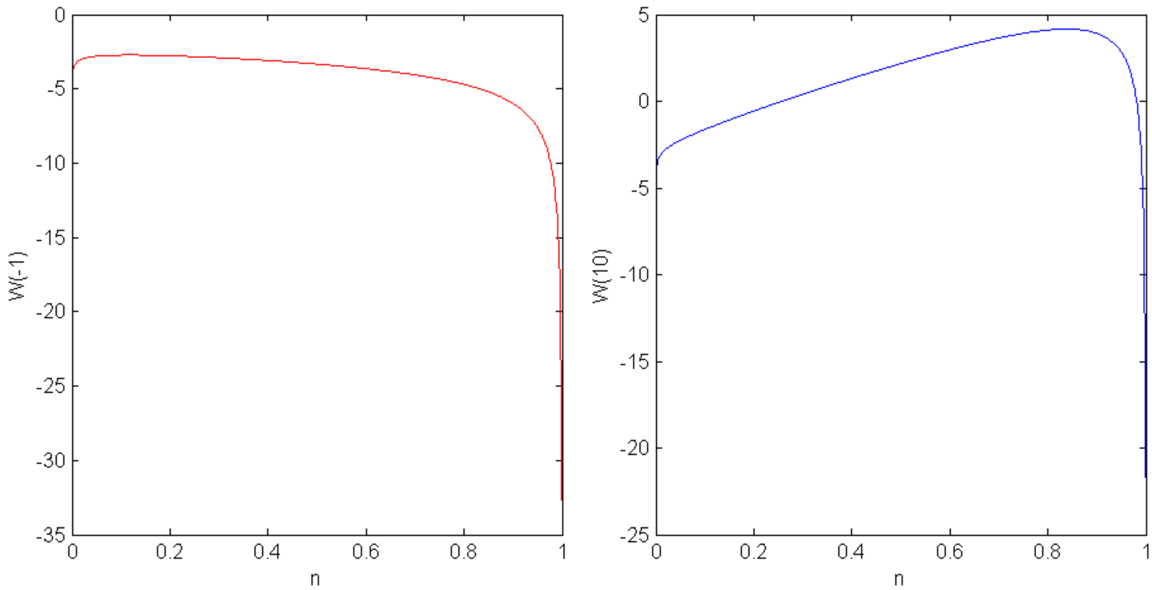


Figure 3: Welfare as a Function of Entrepreneurship with Different Fixed Costs Differential

being employed. This is when the planner would like to make more agents become entrepreneurs. Formally, we have the following lemma.

Lemma 1. *If agents do not prefer either profession ($v_w = v_e$), then $n_{op} = n_{mx}$; if agents prefer entrepreneurship ($v_w - v_e > 0$), then $n_{op} > n_{mx}$; if agents prefer being employed ($v_w - v_e < 0$), then $n_{op} < n_{mx}$.*

Proof. Notice that $f(\frac{1-n_{op}}{n_{op}}) - \frac{1}{n_{op}} f_\ell(\frac{1-n_{op}}{n_{op}})$ is positive if and only if $n_{op} < n_{mx}$ and is negative if and only if $n_{op} > n_{mx}$. If $v_w - v_e > 0$, from (2) we know that $f(\frac{1-n_{op}}{n_{op}}) - \frac{1}{n_{op}} f_\ell(\frac{1-n_{op}}{n_{op}})$ is negative, which means $n_{op} > n_{mx}$. Similarly, we can prove the other parts of the lemma. □

Lemma 2. *The socially optimal entrepreneurship level, n_{op}, is decreasing in $v_e - v_w$.*

The proof is omitted because it is obvious from (2). To sum up, the fixed costs of the two professions could be different so that the first-best allocation must balance output and taste/cost for jobs. If people prefer being entrepreneurs to being workers, a social planner would choose an entrepreneurship level higher than that which maximizes output and vice versa. Furthermore, the higher the (relative) cost of entrepreneurship, the lower the entrepreneurship level that the social planner would choose.

2.2 Timing of the Market Economy

In a pure-currency market economy, agents can hold an object called money. There are potentially two types of transactions: purchasing a consumption good and paying a labor cost. Both types of transactions require money. The evolution of the (real) money balance of workers and entrepreneurs is shown in Figure 3. Workers receive wages after they supply their labor to entrepreneurs and gradually deplete their money holdings by purchasing the goods sold by entrepreneurs until they receive wages again. This process looks like what Baumol (1952) describes. But the same process also implies that the cash holdings of entrepreneurs would gradually increase after they pay wages to workers until they pay workers again. Of course, we have assumed all entrepreneurs have the same production cycle, whereas the reality can be more complicated. Nevertheless, Figure 3 shows that except for measure zero instances of time, both workers and entrepreneurs hold money. Also note that in the background, we should consider that entrepreneurs need money to purchase goods from other entrepreneurs. But since entrepreneurs keep receiving cash from workers, they do not necessarily need to hold cash for consumption purposes right after they pay wages to their workers.

In continuous time, it is hard to capture the discreteness of the production process and wage payments (i.e., output and wage payments are made in bursts). Therefore,

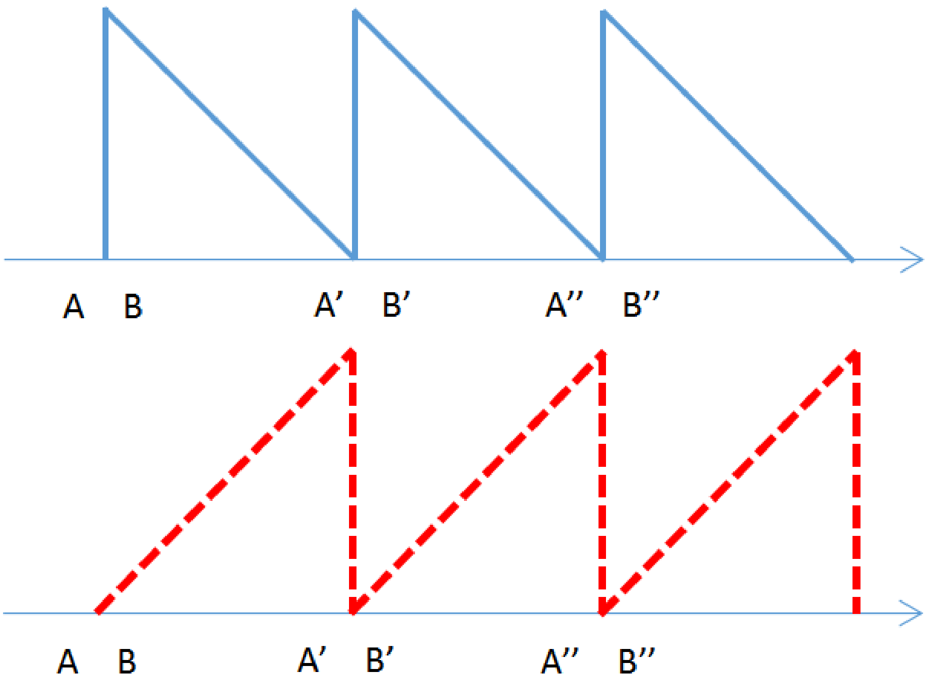


Figure 4: Evolution of Cash Holdings of Workers (Top) and Entrepreneurs (Bottom)

we work in discrete time. This also makes the current work comparable to many previous works in discrete time. But then it is unclear whether we should assume that a period starts with wage payments or ends with wage payments. In Figure 4, we can say that a period ends at point A or B.[7] This is an important and yet seldom discussed question. Because the value of money changes across periods owing to inflation, it matters who carries money across periods. In traditional cash-in-advance models with discrete time, the wage payment happens at the end of a period. This timing might matter for our results, so in this paper, we discuss both settings. We can show that the results are unchanged under both settings. As far as I know, this is the first time such a timing issue has been formally discussed. Below we first presents a version of the model in which a period ends at point A and then study the other version later as robustness check.

2.3 Each Period Begins with Wage Payments

Now we study a version of the model in which workers get paid at the beginning of a period (i.e., a period ends at point A in Figure 4). Thus, it appears that it is entrepreneurs who need to hold money across periods. We start by introducing the market economy with exogenous entrepreneurship to build up some intuition, and then discuss endogenous entrepreneurship and the relationship between inflation and out-

[7] Technically, a period can also end in the middle of two wage payments. But that does not seem like a natural way to define a period and not very comparable to previous studies.

put.

2.3.1 Market Economy with Exogenous Entrepreneurship

The aggregate money supply in the market economy is M, which is controlled by the central bank by lump-sum taxes/transfers T. If necessary, I use the superscript w or e to indicate that a variable belongs to a worker or an entrepreneur. Now we can write the value function for both workers and entrepreneurs:

$$V_t^i(\phi_t m_t) = \max_{x_t^i, y_t^i, b_t^i, m_{t+1}^i} u(x_t^i) + y_t^i - v_i + \beta V_{t+1}^i(\phi_{t+1} m_{t+1}^i) \tag{3}$$

$$\begin{aligned} s.t.\ \phi_t m_{t+1}^i = {} & Y + I_t^i\left(m_t + b_t^i\right) + \phi_t\left(m_t + b_t^i\right) \\ & + \phi_t T_t - x_t^i p_t - y_t^i - b_t^i \phi_t R_t, \end{aligned} \tag{4}$$

$$b_t^i \geq -m_t, \tag{5}$$

where p_t and ϕ_t are the price of good x and that of money in terms of the endowment good y, m_t the money holding to start with in period t (before wage is paid), m_{t+1}^i the planned money holding for next period for profession i, b_t^i the nominal borrowing from the financial market (negative b means lending), R_t the gross nominal interest rate in the financial market, and $I_t^i\left(m_t + b_t^i\right)$ the market income of profession i as a function of the liquidity one can spend during a period, $m_t + b_t^i$. It is important to note that even though a borrower only repays the debt $b_t R_t$ in period $t+1$, it affects m_{t+1} in the budget constraint in period t because m_{t+1} is the planned money holding (net of the repayment of debt).[8] The constraint (5) simply says that one cannot lend more than all of one's money holding. We do not impose a cash constraint for consumption good x for workers, because in addition to the money holding they brought from the previous period, m_t, they receive a wage payment at the beginning of the period, and they can also borrow so that their consumption of x is not constrained by m_t. Entrepreneurs' consumption is not constrained by their money holding m_t because they constantly receive money from other workers (see Figure 4), so they can use their revenue to purchase x from other entrepreneurs. For both types of agents, purchasing consumption goods requires money, but we do not need a cash constraint to describe it.

A worker's market income is independent of how much liquidity he or she has (i.e., $I_t^w\left(m_t + b_t^w\right) = \omega_t$). However, an entrepreneur's market income might depend on the liquidity. Technically, workers have supplied their labor in the previous period and only get paid at the beginning of the current period. We assume that entrepreneurs

[8]One can assume that the borrowing and lending happen at the beginning of a period so that the repayment happens at the beginning of next period. Our definition of m_{t+1} is thus the money holding after a borrower has repaid his debt.

cannot sell the final product unless they have paid the workers. Thus, ℓ_t can be more precisely interpreted as the labor input that an entrepreneur is willing to pay at the beginning of period t. Specifically, an entrepreneur's market income is the solution to the following problem:

$$I_t^e\left(m_t+b_t^e\right) = \max_{\ell_t} p_t f(\ell_t)-\omega_t\ell_t \tag{6}$$

$$s.t. \quad \omega_t\ell_t \leq \phi_t\left(m_t+b_t^e\right), \tag{7}$$

where the constraint says that the production cost can be paid by an entrepreneur's own money or by borrowed money. This is different from the so-called cash-in-advance (CIA) constraint: here, entrepreneurs can finance their production cost by borrowing from the financial market. If they have extra money, they can also lend.

Also note that the labor market and the financial market are assumed to be competitive. The searching and match framework (see Williamson and Wright 2010 for a survey) has become so popular in studies of the effects of long-run inflation that perhaps a few words of justification are needed for my not having used it here. First, as in Rocheteau and Wright (2005), the frictions that make money essential do not preclude price taking for monetary transactions. Second, a nonsearch environment makes the model simpler and helps us to focus on the issue at hand: occupational choices.

To sum up, workers and entrepreneurs differ in that workers derive labor income, whereas entrepreneurs generate revenue from their business, which depends on the liquidity available. Assume Y is large so that we are sure that the solution for y will be interior, which is the interesting case in which the quasi-linear utility helps with tractability. When we plug the budget constraint into the value function and eliminate y, we have

$$\begin{aligned} V_t^i(\phi_t m_t) = & \max_{x_t^i, b_t^i, m_{t+1}^i} u(x_t^i)-x_t^i p_t - v_i + Y + I_t^i\left(m_t+b_t^i\right)+\phi_t T_t \\ & +\phi_t\left(m_t+b_t^i\right)-\phi_t m_{t+1}^i - b_t^i\phi_t R_t+\beta V_{t+1}^i(\phi_{t+1}m_{t+1}^i) \qquad (8) \\ & b_t^i \geq -m_t. \qquad (9) \end{aligned}$$

For workers, the marginal benefit of borrowing is $-\phi_t\left(R_t-1\right)$. Thus, as long as $R_t > 1$, it is optimal for workers to choose the corner solution $b_t^w = -m_t$. In other words, whenever workers hold money at the beginning of the period, they would deposit the cash holdings into a bank, and would not carry them during the period. Then it is clear that $\partial V_t^w(\phi_t m_t)/\partial m_t = \phi_t R_t$.

Entrepreneurs would not carry idle cash during the period neither. Thus, the constraint (7) must hold with equality because otherwise an entrepreneur can deposit idle cash into a bank to earn interest, as would workers. Therefore, we can eliminate ℓ_t in

(6), and the first-order condition for b_t^e becomes

$$\frac{p_t}{\omega_t} f_\ell \left[\frac{\phi_t \left(m_t + b_t^e\right)}{\omega_t} \right] = R_t. \tag{10}$$

Therefore, we have $\partial V_t^w(\phi_t m_t)/\partial m_t = \phi_t p_t f_\ell \left[\phi_t \left(m_t + b_t^e\right)/\omega_t\right]/\omega_t = \phi_t R_t$. The first-order conditions with respect to x_t^i and m_{t+1}^i are

$$u_x(x_t^i) = p_t, \text{ and } \phi_t = \beta \partial V_{t+1}^i(\phi_{t+1} m_{t+1})/\partial m_{t+1} = \phi_{t+1} \beta R_{t+1}, \tag{11}$$

the latter of which simply means that $R_{t+1} = \left(1 + \pi_{t+1}\right)/\beta$, which is the usual Fisher equation. Market clearing for labor and the market good requires the following:

$$\begin{aligned} 1 - n &= n\ell_t & (12) \\ nf(\ell_t) &= x_t. & (13) \end{aligned}$$

The equilibrium of this simple model is the set of $\{x_t^i, \ell_t, m_{t+1}^i\}$ that satisfies the agents' maximization problems and the two market-clearing conditions. We are only interested in the steady-state equilibrium. In this case, all the real variables will be constant. Since neither type of agent holds idle cash, all the money supply is spent by the entrepreneurs to buy labor input. Thus, $\phi_t M_t = (1 - n)\omega_t$, and we know that $\phi_t M_t$ is constant in the steady-state equilibrium, which means that inflation is equal to the growth rate of the money supply. Let $M_{t+1}/M_t = 1 + \pi$. The following proposition summarizes the results with exogenous entrepreneurship.

Proposition 1. *When a period starts with wage payments, if entrepreneurship is exogenous and the individual labor supply is inelastic, then inflation lowers the steady-state real wage and output is unchanged.*

Proof. We can combine the two equations in (11) and the two market-clearing conditions (12)-(13) and have an equation of only one unknown ω, as follows:

$$\frac{1+\pi}{\beta}\omega = u_x[nf(\frac{1-n}{n})]f_\ell(\frac{1-n}{n}). \tag{14}$$

Since n is exogenous, there is only one policy variable: the inflation rate π and one endogenous variable: the real wage ω. It is obvious from the above equations that when π increases, ω decreases. □

The reason is very straightforward: employment and output are determined by the two market-clearing conditions: (12) and (13). As long as n is fixed, so is $1 - n$, and the same applies to the aggregate labor supply and $(1 - n)/n$, the labor employed by an individual entrepreneur. Total output is thus also fixed. Inflation raises the cost of using money, thus lowering the demand for labor because of the Euler equation (11). But since labor supply is fixed, inflation simply lowers the real wage and redistributes wealth from workers to entrepreneurs (everyone receives the same transfer). This result helps us to understand why inflation encourages entrepreneurship if free entry to entrepreneurship is allowed.

2.3.2 Market Economy with Endogenous Entrepreneurship

Now consider endogenous entrepreneurship in a monetary market economy. Assume that at the end of any period, agents can choose an occupation for the next period. Entry to entrepreneurship itself is free.[9] However, when a worker tries to become an entrepreneur in the next period, she needs to acquire working capital this period (or borrow it). Specifically, the value function of agents is now

$$\begin{aligned} V_t^i(\phi_t m_t) &= \max_{x_t^i, b_t^i} u(x_t) - x_t p_t - v_i + Y + I_t^i \left(m_t + b_t^i \right) + \phi_t \left(m_t + b_t^i \right) - b_t^i \phi_t R_t \\ &\quad + \phi_t T_t + \max\{ \max_{m_{t+1}^w} -\phi_t m_{t+1}^w + \beta V_{t+1}^w(\phi_{t+1} m_{t+1}^w), \\ &\quad \max_{m_{t+1}^e} -\phi_t m_{t+1}^e + \beta V_{t+1}^e(\phi_{t+1} m_{t+1}^e) \}. \end{aligned} \tag{15}$$

Free entry would require the two alternatives inside the second max operator to be equalized. This means that the payoffs of the two professions should be the same for every period. Using the fact that $b_{t+1}^w = -m_{t+1}^w$, $\omega_t \ell_t = \phi_t \left(m_t + b_t^e \right)$, and the definition of $I_t^i \left(m_t + b_t^e \right)$, we have the following equation:

$$\begin{aligned} & \max_{m_{t+1}^e, b_{t+1}^i} \{ -\phi_t m_{t+1}^w + \beta [\omega_{t+1} + m_{t+1}^w \phi_{t+1} R_{t+1} - v_e] \} \\ = & \max_{m_{t+1}^e, b_{t+1}^i} \{ -\phi_t m_{t+1}^e + \beta [p_{t+1} f \left(\ell_t \right) - \omega_{t+1} \ell_{t+1} R_{t+1} + m_{t+1}^e \phi_{t+1} R_{t+1} - v_e] \}. \end{aligned} \tag{16}$$

In steady state, $n_t = n$, $n \phi_t m_t^e = \omega(1-n)$, and $\phi_t m_{t+1}^e = \phi_{t+1} m_{t+1}^e (1+\pi)$. After some algebra, we can write

$$\omega - v_w = -\frac{1+\pi}{\beta} \frac{1-n}{n} \omega + u_x [n f(\frac{1-n}{n})] f(\frac{1-n}{n}) - v_e. \tag{17}$$

The left-hand side is the wage minus fixed costs for a worker. On the right-hand side, the first term is the labor cost of production for an entrepreneur who hires $(1 - n)/n$ unit of labor. The reason for the $(1+\pi)/\beta$ term is because the production cost is associated with the nominal interest rate, as shown in equation (10), and is thus

[9] Other monetary models, such as Williamson (1994) and Rocheteau and Wright (2005), also consider endogenous participation or entry. The entry to entrepreneurship discussed here is different in that one more entrepreneur means one additional production unit and also one less worker.

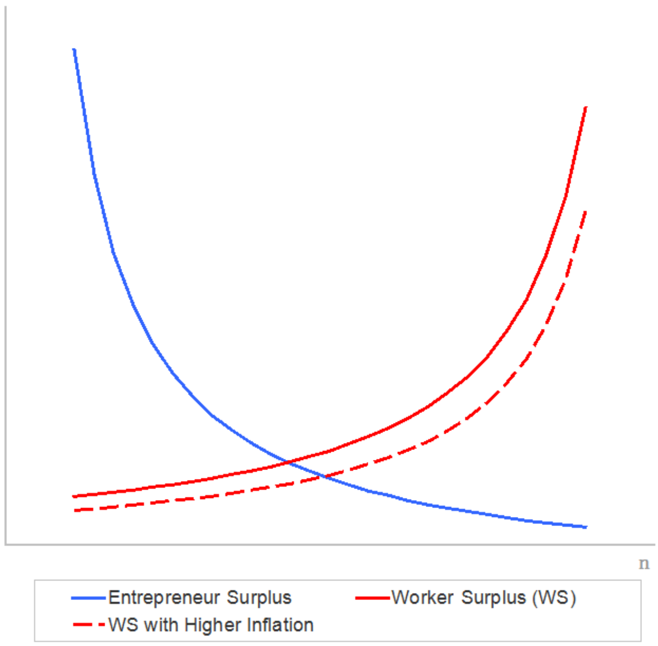


Figure 5: Effects of Higher Inflation

related to the steady-state inflation rate. The second term on the right-hand side is the price of the market good in terms of the endowment good y times the output of an entrepreneur. Note that $u_x(x) = p$ according to (11). When we eliminate ω using the simplified Euler equation (14), we can reduce the steady-state equilibrium system into one equation and one unknown, n:

$$u_x[nf(\frac{1-n}{n})][f(\frac{1-n}{n}) - \frac{1}{n}f_\ell(\frac{1-n}{n}) + f_\ell(\frac{1-n}{n})(1 - \frac{\beta}{1+\pi})] + v_w - v_e = 0. \quad (18)$$

When $u(x) = \ln(x)$ and $f(\ell) = Z\ell^\alpha$, the above equation can be further simplified:

$$\alpha\frac{1}{1-n}\frac{\beta}{1+\pi} - v_w = (1-\alpha)\frac{1}{n} - v_e. \quad (19)$$

The left-hand side is worker surplus, and the right-hand side is entrepreneur surplus, as shown in Figure 5. In an equilibrium with free entry, agents would be indifferent between the two professions. The two types of surplus would be equalized. We can see that in this particular case, the worker surplus is decreasing in π and increasing in n, whereas entrepreneur surplus is decreasing in n. Thus, an increase in π will shift the worker surplus curve down so as to increase n in the steady-state equilibrium. Also note that an increase in $v_w - v_e$ also increases n. These two results can be proven for the general CRRA utility function as well,[10] as shown by the following proposition.

[10]When the CRRA parameter, ρ, is smaller than one, we still have worker surplus (WS) increasing in n and entrepreneur surplus decreasing in n. But if $\rho > 1$, then both surplus measures will be U-shaped. But we can show that the slope of WS is still higher than entrepreneur surplus (ES) in equilibrium (see

Proposition 2. *Let a period start with wage payments. Assume any CRRA utility function $u(x) = x^{1-\rho}/(1-\rho)$, with $\rho > 0$, and any production function $f(\ell) = \ell^{\alpha}$, with $\alpha \in (0,1)$. If entrepreneurship is endogenous and labor supply is inelastic, then entrepreneurship, n, is increasing in both inflation and $v_w - v_e$ in the steady state.*

Proof. See Appendix A. □

Here is a concise description of the intuition. Inflation is a tax on cash holdings. Even though entrepreneurs can borrow to finance their production, they need to compensate the lenders with a nominal interest rate that is consistent with the steady-state inflation rate. The nominal interest rate is what matters because (as mentioned above) even though a borrower only repays the debt $b_t R_t$ in period $t+1$, m_{t+1} is already affected in the budget constraint in period t. Remember m_{t+1} is the planned money holding (net of the repayment of debt). In other words, even though the debt is repaid at the beginning of next period, the borrowers need to put aside the money needed in the current period. Thus the cost of borrowing is the nominal interest rate.

Higher inflation, and thus the nominal interest rate, raise the cost of production for entrepreneurs. This has negative first-order effects on the demand for labor and thus real wages. Entrepreneurs are also worse off because the cost of employing a worker, $(1+\pi)\omega/\beta$, could still be higher. But thanks to the Cobb-Douglas production function, labor cost is a constant α fraction of output. Note that workers' real wage income is the labor cost net of the inflation tax. Thus, inflation has first-order effects on real wages but only general equilibrium effects on entrepreneurs' business income. To sum up, higher inflation hits workers harder than entrepreneurs, thereby pushing workers into entrepreneurship.

2.3.3 Optimal Inflation and Implications on Output

As for output and welfare, the policy implications are subtle. The effects of inflation can be summarized by the following proposition.

Proposition 3. *Let a period start with wage payments. Assume any CRRA utility function $u(x) = x^{1-\rho}/(1-\rho)$, with $\rho > 0$, and any production function $f(\ell) = \ell^{\alpha}$, with $\alpha \in (0,1)$. With inelastic labor supply and endogenous entrepreneurship, the Friedman rule (i.e., $1+\pi = \beta$) attains the first best by setting the socially optimal entrepreneurship (i.e., $n = n_{op}$). In the steady state, inflation always decreases welfare, but it increases output if (a) $v_e > v_w$ and (b) $n < \tilde{n}$. If either condition is violated, then inflation decreases output.*

Proof. Compare the planner's first-order conditions, (2), and the equilibrium condition of the market economy, (18). We can see that when $(1+\pi)/\beta = 1$, the two conditions

Proposition 2).

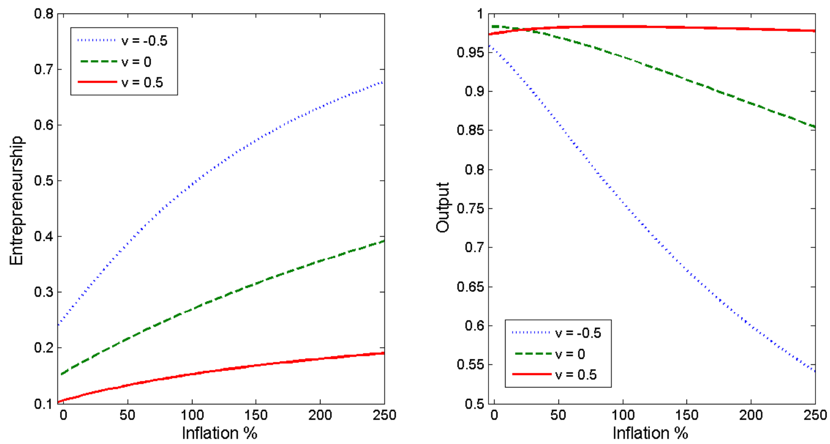


Figure 6: Effects of Inflation on Entrepreneurship and Output ($v = v_e - v_w$)

coincide. This means that n equals the socially optimal n_{op}. When $(1+\pi)/\beta$ increases from 1, n increases from n_{op}, according to Proposition 2. This would lower welfare, since the social welfare function is concave and is maximized at n_{op}.

If $v_w - v_e$ is negative, then we know that n_{op} is smaller than n_{mx}, which maximizes output. Since entrepreneurship is increasing in inflation, and output is increasing in n if $n < n_{mx}$ and decreasing in n if $n > n_{mx}$, we know that inflation raises output if $n < n_{mx}$ and decreases output if $n > n_{mx}$. On the other hand, if $v_w - v_e \geq 0$, then $n_{op} > n_{mx}$, so any increase in inflation would lower output. □

Figure 6 shows a simulated example in which we consider three values of v, which is short for $v_e - v_w$. In the left panel, we confirm the result in Proposition 1: inflation always encourages entrepreneurship. Right panel shows how the effects of inflation on output depend on the fixed cost differential. If v is nonpositive (the fixed cost for entrepreneurs is lower), then inflation always decreases output. If v is positive (the fixed cost for entrepreneurs is higher), then inflation first increases and then decreases output.

What is the intuition behind this effect? We need to first note that output is solely a function of entrepreneurship, n. As shown in Proposition 3, when $v_e = v_w$, the socially optimal entrepreneurship is the same as the output-maximizing one. Proposition 2 says that the socially optimal entrepreneurship is obtained with the Friedman rule. Any increase in entrepreneurship would decrease output, and that is what inflation does with $v = 0$. Now, if v is negative ($v_w > v_e$), that means the social planner would choose an entrepreneurship level above that which maximizes output. This is shown in the left panel of Figure 6: entrepreneurship is higher when v is negative. Therefore, increases in inflation further raise entrepreneurship and decrease output. On the other hand, if v is positive ($v_w < v_e$), that means the social planner would choose an entrepreneurship level below that which maximizes output. Now inflation first raises output, then after n has already reached the output-maximizing level, a further increase in inflation reduces output. We can see that with $v = 0.5$, output peaks around 100% inflation, which corresponds to an entrepreneurship level that is the same as with $v = 0$ around the Friedman rule.

Now that the result depends crucially on $v_w - v_e$, it is useful to think about what these parameters reflect in the real world. First, v_w and v_e shall include the non-pecuniary costs or disutility from work for the two occupations. Generally, these non-pecuniary costs are different for the two professions. For example, Hamilton (2000) finds that "entrepreneurs may trade lower earnings for the non-pecuniary benefits of business ownership" (p. 605). Entrepreneurship usually provides more flexible working hours and other benefits, such as less need to take orders from others. Second, v_e can also include some nonmarket fixed costs of doing business. In countries where entrepreneurs have to bribe government officials to conduct business, or if doing business is risky because of weak law enforcement and so on, v_e could be very high. Third, v_w could also reflect the working conditions of workers. Although the nonpecuniary costs of the two professions seem more universal, the fixed costs of doing business could be very different across countries.

What is special about Proposition 2 is not that it states a theoretical possibility that the relationship between inflation and output could be nonmonotonic. It is the fact that this theory tells us about a simple and testable condition for the nonmonotonic effect to exist: the sign of the occupational fixed cost differential. The business cost can vary greatly across countries. Thus, Proposition 2 opens up the possibility that the effects of long-run monetary policy could be very different across countries.

Further, if the nonmonotonic effects exist, the threshold above which inflation negatively affects output could also be different across countries, as stated in the following corollary.

Corollary 1. *If $v_e - v_w > 0$, an increase in $v_e - v_w$ raises the threshold, π_{mx}, above which inflation negatively affects output.*

The proof is omitted. For illustrative purposes, consider the case of log utility, so equilibrium is described by equation (19). Suppose given a positive value of $v_e - v_w$, inflation rate π_1 makes the equilibrium n equal to n_{mx}, which maximizes output. Then π_1 is now at the threshold above which inflation would reduce output. Now suppose that $v_e - v_w$ increases. Then according to equation (19), in order to achieve the same n, we must have a higher inflation rate, $\pi_2 > \pi_1$.

According to this result, high-business-cost (HBC) economies, such as China and India, would have a higher threshold above which inflation negatively affects output. This is a novel prediction and arguably quite relevant for our understanding of inflation. Relatedly, Lopez-Villavicencio and Mignon (2011) show that the thresholds above which inflation negatively affects output growth are indeed higher for developing economies than for developed economies.

Lastly, this paper contributes to the positive studies of inflation. The pioneering work of Kydland and Prescott (1977) and Barro and Gordon (1983) stimulate a large literature, as surveyed by Berger et al. (2001). We have known that characteristics of the central banks, especially their independence, matter for the cross-country differences in inflation rates. Here we provide a new explanation. According to the theory in this study, an HBC country might choose an inflation rate that is different from that in a low-business-cost (LBC) country because they face different long-run tradeoffs between inflation and output. This is consistent with the observation that many emerging market economies seem to have a greater "tolerance" for inflation.

2.4 Each Period Ends with Wage Payments

2.4.1 Environment and Individual Optimization

Here we study another version of the model in which each period ends with wage payments (i.e. point B's in Figure 4). Since we redefine a period, we need to make many changes in the mathematical descriptions. Specifically, four modifications are made. First, we remove the endowment good y to ignore the transition and the formal problem of occupational choices. Instead, when we study endogenous entrepreneurship, we shall pick the n to make sure the two professions have the same payoffs. Second, we open a financial market for agents to borrow and lend at the end of a period. This is because what happens at the beginning of a period in the previous version of the model now happens at the end of a period. So naturally entrepreneurs have a strong incentive to access such a financial market. Notice that unlike the debt incurred at the beginning of a period, the debt incurred at the end of a period needs to be included in the state variables of the value functions. Third, we also allow agents to borrow or lend at the beginning of a period, which is attractive to workers (this is especially true since we have removed the endowment good). Both markets are open because the agents in the model need them. Last, the lump-sum transfers/taxes by the central bank happen at the beginning of a period. This is simply to be consistent with the previous version: they still happen after wage payments and can still be used for purchasing consumption goods. Now it is better to describe the problem of the two types of agents separately. The value function of a worker is as follows:

$$V_t^w(m_t^w, d_{t-1}^w) = \max_{x_t^i, y_t^i, b_t^i, \ell_t^i, d_t^i, m_{t+1}^i} u(x_t^w) - v_w + \beta V_{t+1}^w(m_{t+1}^w, d_t^w) \tag{20}$$

$$\begin{aligned} s.t.\ \phi_t m_{t+1}^w &= \omega_t + \phi_t\left(m_t^w + b_t^w\right) + \phi_t T_t - x_t^w - b_t^w \phi_t R_t \\ &\quad - \phi_t d_{t-1}^w R_{t-1}^d + \phi_t d_t^w \end{aligned} \tag{21}$$

$$x_t^w \leq \phi_t\left(m_t^w + b_t^w\right) + \phi_t T_t \tag{22}$$

$$b_t^w \geq -m_t^w - T_t, \tag{23}$$

where d_{t-1}^w is debt incurred at the end of period $t-1$, R_{t-1}^d the corresponding nominal interest rate, b_t^w the debt incurred at the beginning of period t, R_t the corresponding nominal interest rate, ϕ_t the value of money in terms of consumption good x (it was in terms of endowment good y in the previous version), ω_t real wage in terms of the consumption good x, and T_t the lump-sum transfers/taxes imposed by the central bank. Positions of both types of debt can be negative (lending). The cash constraint (22) says workers can use their own money, borrowed money, and the transfers received from the central bank to make purchases. Workers face this constraint because they receive their income at the end of a period. As in the previous version, the cash constraint must always be binding because otherwise workers can always borrow less or deposit more. Using the first two constraints and the envelope condition, $\partial V_t^w / \partial m_t^w = \phi_t u_x\left(x_t^w\right)$, we can eliminate x_t^w and m_{t+1}^w and write the first-order conditions with respect to b_t^w and d_t^w as

$$\phi_t u_x\left(x_t^w\right) = \beta \phi_{t+1} u_x\left(x_{t+1}^w\right) R_t \tag{24}$$

$$\phi_{t+1} u_x\left(x_{t+1}^w\right) = \beta \phi_{t+2} u_x\left(x_{t+2}^w\right) R_t^d. \tag{25}$$

These conditions are intuitive: if a worker chooses to borrow more at the beginning of a period, b_t^w, the marginal benefit is more consumption today, and the marginal cost is less consumption tomorrow because that is when the repayment happens. If a worker chooses to borrow more at the end of a period, d_t^w, the marginal benefit is more consumption in period $t+1$, and the marginal cost is less consumption in period $t+2$. This is because the repayment happens at the end of $t+1$ and reduces the money that can be used for $t+2$ consumption. If x_t and π_t are constant, then $R_t = R_t^d = (1+\pi)/\beta$.

Next, for an entrepreneur, we need to add one more state variable: the quantity of labor input paid in the last period; it matters how much goods can be sold in the current period. The value function for an entrepreneur is as follows:

$$V_t^e(m_t^e, \ell_{t-1}^e, d_{t-1}^e) = \max_{x_t^i, b_t^i, \ell_t^i, d_t^i, m_{t+1}^i} u(x_t^e) - v_e + \beta V_{t+1}^e(m_{t+1}^e, \ell_t^e, d_t^e) \tag{26}$$

$$s.t.\ \phi_t m_{t+1}^e = f\left(\ell_{t-1}^e\right) + \phi_t\left(m_t + b_t^e\right) + \phi_t T_t - x_t^e \tag{27}$$

$$- b_t^e \phi_t R_t - \omega_t \ell_t^e - \phi_t d_{t-1}^e R_{t-1}^d + \phi_t d_t^e, \tag{28}$$

$$x_t^e \leq \phi_t\left(m_t^e + b_t^e\right) + f\left(\ell_{t-1}^e\right) + \phi_t T_t \tag{29}$$

$$b_t^e \geq -m_t - T_t, \tag{30}$$

where the constraint (29) says an entrepreneur can use the current revenue to make purchases. It is clear from Figure 4 that entrepreneurs are constantly receiving income in terms of money from workers during a period. Being an entrepreneur thus has an important advantage over being a worker: an entrepreneur's consumption is not limited by the liquidity carried over from the preceding period and that acquired at the beginning of a period. The timing assumption of this version (about when prices change) is important for this. But another fact is also necessary: entrepreneurs receive income constantly during a period whereas workers receive income once in a period.[11]

Assuming $f\left(\ell_{t-1}^e\right) > x_t^e$, then it is clear that the optimal $m_{t+1}^e = 0$. Using the budget constraint (27), we can eliminate ℓ_t, and then using the envelope condition of ℓ_t: $\partial V_t^e / \partial \ell_{t-1}^e = f_\ell\left(\ell_{t-1}\right) \frac{1}{\omega_t} \beta \partial V_{t+1}^e / \partial \ell_t$, we can write the first-order condition with respect to x_t^e and d_t^e as follows:

$$u_x\left(x_t^e\right) \omega_t = \beta f_\ell\left(\ell_t\right) u_x\left(x_{t+1}^e\right) \tag{31}$$

$$\frac{\phi_t}{\omega_t} f_\ell\left(\ell_t\right) = \phi_{t+1} R_t^d, \tag{32}$$

where the first condition equates the marginal cost and benefit of obtaining one unit of labor input in terms of consumption, and the second condition does the same for end-of-period borrowing in terms of output. Since we know from the workers' problem that in a steady state, the Fisher equation holds with $R_t^d = (1+\pi)/\beta$, we know inflation does not distort the production of entrepreneurs since $\omega = \beta f_\ell(\ell)$. This is a different finding from the previous version.

To consider endogenous entrepreneurship, first assume $v_w - v_e = 0$, so that we want to set the income of the two professions to be the same. The following condition says that working as a worker and as an entrepreneur in period t should give the same payoff in terms of the consumption good in period t:

$$\beta \frac{\omega_t}{\phi_t} \phi_{t+1} + \beta \phi_{t+1} T_{t+1} - v_w = -\omega_t \ell_t + \beta f\left(\ell_t\right) + \beta \phi_{t+1} T_{t+1} - v_e, \tag{33}$$

[11] Of course, we do not allow entrepreneurs to deposit their constant cash flows during a period. That seems hard to capture in models with discrete time. Studying the impact of this is left to future studies.

where we have used four facts: first, $v_w = v_e$; second, one unit of the consumption good in period $t+1$ is worth β units of that in period t; third, a worker receives a wage payment at the end of a period, so he or she can only acquire the consumption good the next day and has to be burdened with the inflation tax; fourth, an entrepreneur pays wages today and receives revenue tomorrow but does not need to hold money across periods. This condition highlights the trade-off in switching occupations in terms of consumption goods. Because agents have access to a perfect financial market, the above equation also measures income from a permanent income perspective. When $v_w = v_e$, agents only care about permanent income. It is important to note that equation (33) shows that inflation would have first-order effects on workers and only general equilibrium effects on entrepreneurs. The condition also highlights another fact: inflation act like a tax on workers and redistributes part of their income to everyone. So effectively, inflation transfers income from workers to entrepreneurs.

When $v_w \neq v_e$, switching occupations involves some transition problems. We can imagine two agents born at the beginning of period t with the same endowment e. One of them chooses to work as a worker, and the other chooses to be an entrepreneur. The optimal consumption plan of the entrepreneur is more complicated: it requires spreading the initial labor cost paid in period t over a long period of time if the utility function is concave. Loading all of the cost in period t at once is just too costly. This is how the endowment good with quasi-linear utility becomes useful in the previous version. However, there are two cases in which equation (33) is still valid: (a) if utility function is linear $u(x) = x$; (b) if the costs of v_w and v_e are paid in consumption goods.

2.4.2 Steady-State Equilibrium

The labor market clearing condition is the same as (12), but the consumption goods market clearing condition is now different (because we have removed the endowment good):

$$(1 - n_t)\, x_t^w + n_t x_t^e = n f(\ell_t), \tag{34}$$

where in general $x_t^w \neq x_t^e$. We assume the money received by workers as wage payments cannot be loaned out immediately. Then at the end of a period, all the money stock would be used for wage payments (i.e.,$\phi_t\,(M_t + T_t) = (1 - n_t)\,\omega_t$). Here M_t is the money stock at the beginning of a period before central bank interventions. Since $M_{t+1} = M_t + T_t$, we know that if the central bank chooses $\pi_t = T_t/M_t$ to be constant, we will then have a constant inflation rate.

In a steady-state equilibrium, all real variables will be constant, including ω_t and thus $\phi_t\,(M_t + T_t)$. Thus, we have $\phi_t T_t = \pi\,(1 - n)\,\omega / \,(1 + \pi)$. We can write a steady-state version of equation (33) as follows:

$$\beta\frac{1}{1+\pi}\omega+\beta\frac{\pi}{1+\pi}\left(1-n\right)\omega-v_w=-\omega\ell+\beta f\left(\ell\right)+\beta\frac{\pi}{1+\pi}\left(1-n\right)\omega-v_e, \quad (35)$$

which makes it clear that steady-state inflation taxes workers and redistributes it to everyone. Since $\ell=\left(1-n\right)/n$, and $\omega=\beta f_\ell\left(\ell\right)$, if further $f\left(\ell\right)=\ell^\alpha$, we can write a steady-state version of equation (33) as follows:

$$\beta\frac{\beta}{1+\pi}\alpha\left(\frac{1-n}{n}\right)^{\alpha-1}-v_w=\beta\left(1-\alpha\right)\left(\frac{1-n}{n}\right)^{\alpha}-v_e. \quad (36)$$

The left-hand side is increasing in n, ranging from 0 to ∞ as n increases, whereas the right-hand side is decreasing in n, ranging from ∞ to 0 as n increases. So we know there is a unique steady-state equilibrium, and clearly an increase in π would raise entrepreneurship. We thus state the following proposition without proof.

Proposition 4. *Let a period end with wage payments. Assume any production function $f(\ell)=\ell^\alpha$, with $\alpha\in(0,1)$. If entrepreneurship is endogenous and labor supply is inelastic, then there exists a unique equilibrium. Entrepreneurship, n, is increasing in the inflation rate in the steady state if one of the following conditions is satisfied: (a) $v_w=v_e$; (b) $v_w\neq v_e$ and $u\left(x\right)=x$; (c) $v_w\neq v_e$, and v_w and v_e are paid in terms of consumption goods. In addition, entrepreneurship is increasing in v_w-v_e if (b) or (c) is satisfied.*

The above proposition is similar to but different from Proposition 2. Note that in the previous version of the model, wage payment are paid at the beginning of a period. No matter who carries the money across periods, the nominal interest rate adjusts for the inflation cost. Higher inflation means lower demand for labor; thus, inflation has first-order effects on workers and transfers some of their income to entrepreneurs. In the current version, wage payments are paid at the end of a period. Workers do not have new income during a period, so they face the nominal interest rate, which accounts for inflation. However, because entrepreneurs constantly receive cash from buyers of their goods (see Figure 4), they do not need to prepare cash at the beginning of a period OR borrow liquidity for their own consumption. Inflation is thus a tax on workers but not on entrepreneurs. Again, inflation transfers some of the income of workers to entrepreneurs. Of course, if one does not allow entrepreneurs to make purchases using current revenue, then things would be different. But our assumption is consistent with what Figure 4 describes: everyone consumes gradually so entrepreneurs receive income constantly, but workers get paid once in a period. Such formulation avoids the assumption that "the husband works and the wife shops", which is used to justify the cash-in-advance constraint.

Next, we want to establish the counterpart of Proposition 3. Before doing that, we need to reexamine the planner's problem because we have redefined a period. In the previous version of the model, we actually let the fixed cost of occupation be paid with wage payments; that is, in period t, a worker supplies labor, and then in period $t+1$, she/he incurs disutility v_w, gets paid, and consumes (and similarly for entrepreneurs). In the current setup, a worker incurs disutility v_w and gets paid in the same period, but consumption comes one period later. Thus, the social planner's problem is to maximize the following object function in the steady state:

$$\max_n \beta u\left[nf\left(\frac{1-n}{n}\right)\right] - nv_e - (1-n)\,v_w, \tag{37}$$

which implies that the socially optimal entrepreneur should satisfy

$$\beta u_x[nf(\frac{1-n}{n})][f(\frac{1-n}{n}) - \frac{1}{n}f_\ell(\frac{1-n}{n})] = v_e - v_w. \tag{38}$$

If $u(x) = x$ and $f(\ell) = \ell^\alpha$, then it is the same as (36) if the Friedman's rule is implemented: $1+\pi = \beta$. We can thus state the following proposition and its corollary without proof:

Proposition 5. *Let a period end with wage payments. Assume any production function $f(\ell) = \ell^\alpha$, with $\alpha \in (0,1)$. Suppose one of the following conditions is satisfied: (a) $v_w = v_e$; (b) $v_w \neq v_e$ and $u(x) = x$; (c) $v_w \neq v_e$, and v_w and v_e are paid in terms of consumption goods. With inelastic labor supply and endogenous entrepreneurship, the Friedman rule (i.e., $1+\pi = \beta$) attains the first best by setting the socially optimal entrepreneurship (i.e., $n = n_{op}$). In the steady state, inflation always decreases welfare, but it increases output if (a') $v_w - v_e < 0$ and (b') $n < \tilde{n}$. If either condition is violated, then inflation decreases output.*

Corollary 2. *If $v_e - v_w > 0$, an increase in $v_e - v_w$ raises the threshold, π_{mx}, above which inflation negatively affects output.*

These results again states that the Friedman rule restores efficiency and that the relationship between long-run inflation and output critically depends on the level of business cost. Perhaps surprisingly, it does not matter how we treat the timing assumption. Our theory thus also provides a micro foundation for why and how money is used by a society: what is important is to recognize that agents consume gradually and wage payments happen once in a period. The economy thus has a natural asynchronization of expenditure and revenue, which motivates the use of money.

2.5 Discussion

Borrowing

In cash-in-advance models, agents are forbidden from borrowing liquidity when they need it. Steady-state inflation is a tax on cash holdings and thus can have real effects.

One might think that once we allow agents to borrow and lend when they need liquidity, then inflation should have no real effects. However, in the above theoretical analysis, we do allow agents to borrow and lend, and yet inflation still can have real effects. This is because the nominal interest rates correctly compensate the lenders for the cost of carrying money across periods due to inflation. This is related to a point made by Kohn (1981): since borrowing and lending net to zero, the money borrowed by one person to evade the "cash constraint" represents a corresponding tightening of the constraint of the lender. Inflation is still a tax. In the model of this paper, those who use money for its liquidity function still have to pay the inflation tax. Now is just now embedded in the nominal interest rate, which can be regarded as the marginal cost of using liquidity provided by money. That is why the famous Friedman rule wants to set the nominal interest rate to zero. However, in other models where agents do not need the liquidity provided by money, the above argument is no longer true and we have the classical dichotomy. It is then only the real interest rate and not the nominal interest rate that matters for the real allocation. But given in real world we do need the liquidity provided by money, should we use the cashless economy as our benchmark and consider the effects of inflation/nominal interest rate in monetary models as an artifact? It seem equally plausible, if not more, to regard the pure-currency economy as a benchmark and think of the superneutrality of money as a result of the oversimplifying cashless paradigm. A cashless model can be applied in other areas of macroeconomics if one takes the monetary policy as given. But it is exactly when we study monetary policies that we should pay attention to money and nominal interest rates.

Different Timing Assumption

In the first timing assumption, entrepreneurs are the borrowers of money at the beginning of a period. They need to compensate the lenders for the inflation tax. But since they borrow the money to purchase labor, the tax burden is then imposed on workers. If the individual labor supply is inelastic, then all of the inflation tax is paid by workers. In equilibrium, inflation has a first-order effect only on workers' real wage. Once the free choice of occupations is allowed, a higher inflation rate will induce more people to become entrepreneurs instead of workers.

In the second timing assumption, workers are the borrowers of money and thus bear the inflation tax. But entrepreneurs are also buyers, and yet because they receive income constantly during a period, they do not need to borrow money at the beginning of the period. In a sense, entrepreneurs carry real goods across periods for their future consumption and thus avoid the inflation tax. It is this differential effect of inflation that induces higher entrepreneurship in the steady state. Of course, the key condition for this result is that workers receive their income at once and consume gradually during a period, whereas entrepreneurs receive their income gradually during a period and pay workers at once.

The assumption of inelastic labor supply is for both tractability and consistency with previous works in the entrepreneurship literature (for a survey see Quadrini 2009). In reality, workers do face "hours constraints" set by firms, that is, at a given wage, many workers cannot freely choose the number of hours they work (see, e.g., Martinez-Granado 2005 and Chetty et al. 2011). On the other hand, the evolution of income and payment in the two professions are like what we described. Thus, we think it is likely that both mechanisms are at work in reality. Because both mechanisms point to the same conclusion, this gives us more confidence in the effects of long-run inflation on entrepreneurship.

More Implications

Because previous empirical studies have ignored the BC, this may help us to understand the existing mixed empirical evidence on the effects of long-run inflation on output/growth. For example, despite some empirical studies that find that higher inflation could be beneficial at low levels of inflation, especially in developing countries, Berentsen et al. (2011) find a monotonically upward-sloping long-run Phillips curve for many developed countries. These are all consistent with the theory of this paper: it depends on the BC. As Panel (a) of Figure 1 shows, countries with more advanced economies tend to have lower BC. Further more, the threshold of this nonmonotonicity is increasing in the BC and thus is different across countries. This is consistent with the observation that many emerging market economies have a greater tolerance for inflation. But it also suggests that detecting the nonmonotonic effect empirically can be more complicated than it appears.

Our model is also related to the positive theory of inflation. The pioneering work of Kydland and Prescott (1977) and Barro and Gordon (1983) stimulates a large literature as surveyed by Berger et al. (2001). We have known that characteristics of the central banks, especially their independence, matter for the cross-country differences in inflation rates. This paper enhances our understanding of the different inflation rates across countries in two ways. First, according to the theory, an HBC country might choose an inflation rate that is different from that in an LBC country because they face qualitatively different long-run trade-offs between inflation and output. Second, since the threshold of nonmonotonicity is increasing in the BC, the HBC countries with higher business costs might tend to choose higher long-term inflation rates if they want to maximize output.

3 Evidence

The theory in this paper has three novel predictions: (a) higher long-term inflation leads to more entrepreneurship; (b) long-term inflation and output should be negatively related in the LBC countries, but the relationship is hump-shaped in the HBC countries; and (c) in the HBC countries, the threshold of nonmonotonicity is increasing in the business cost. There are many reasons why empirically testing (b) is hard, especially with (c). But as long as (a) is true and that output is hump-shaped in entrepreneurship, then (b) and (c) follows naturally. Therefore, next we confront our main prediction (a) with the data.

3.1 Data and Empirical Strategy

Our entrepreneurship data are from the Global Entrepreneurship Monitor (GEM), which covers yearly cross-country series of entrepreneurship from 2001 to 2014. Since we need lagged entrepreneurship as our regressors, our sample period is from 2002 to 2014. Because our theory is about the steady-state relationship of inflation and entrepreneurship, we focus on established entrepreneurship. These entrepreneurs have hired workers and paid salary for more than 42 months. We further adjust for the size of the labor force so that our measure of established entrepreneurship is a percentage of the labor force, which is denoted as EELF. We use the average inflation rate of the past five years as our measure of long-term inflation rates. This also smooths out the effects of the high-frequency movement of inflation. Table 1 describes the summary statistics of the samples used in Tables 2-4, whereas those in the sample used in Tables 5 and 6 are described in Appendix Table B1 to save space in the main text. The definitions for other variables are introduced in the notes of the table that uses them. For a complete list of definitions and sources for our variables, please see Appendix B.

We use panel data regressions to explore the relationship between long-term inflation and EELF.[12] Our baseline specification is as follows:

$$EELF_{it} = \gamma \pi_{it-1}^{5y} + \eta EELF_{it-1} + \boldsymbol{X}'_{it-1}\boldsymbol{\Gamma} + \mu_i + \delta_t + \epsilon_{it}, \tag{39}$$

where π_{it-1}^{5y} is the average inflation rate of the past five years (not including the current year), $\boldsymbol{X}_{it-1}$ a vector of control variables, μ_i the time-invariant country fixed effects, δ_t the common time fixed effects, and ϵ_{it} the error term. The parameter γ is thus our main focus. We include the lagged dependent variable as a current control because it is natural to expect some persistency in the stock of established entrepreneurship. The country fixed effects are included to reduce the endogeneity problem. It has been

[12]Another approach, adopted by Bullard and Keating (1995), requires a relatively long time series of data.

Table 1 : Descriptive Statistics				
	Mean	Std. Dev.	Obs	Countries
Panel A (Table 2)				
Established entrepreneurship in labor force	0.103	0.063	465	74
Inflation 5-yr moving average$_{t-1}$	4.232	3.569	465	74
Panel B (Table 2)				
Established entrepreneurship in labor force	0.099	0.057	363	62
Inflation 5-yr moving average$_{t-1}$	3.916	3.266	363	62
Panel C (Table 3)				
Inflation 5-yr moving average$_{t-1}$	4.399	3.616	411	74
Age structure$_{t-5}$	12.148	4.986	411	74
Panel D (Table 4)				
Established entrepreneurship in labor force	0.107	0.065	411	74
Inflation 5-yr moving average$_{t-1}$	4.399	3.616	411	74
Growth of GDP per capita$_{t-1}$	1.959	3.545	411	74
Openness$_{t-1}$	67.493	46.785	411	74
Tertiary Education Enrollment$_{t-1}$	60.976	20.527	348	64
Financial Depth$_{t-1}$	113.783	72.204	406	73

Notes: Values are averages during the sample period, with standard deviations also reported. Panel A refers to the sample in Table 2, columns 1 and 2; Panel B refers to the sample in Table 2, column 3; Panel C refers to the sample in Table 3, columns 1, 2, and 3; Panel D refers to the sample in Table 5. Inflation 5-yr MA is the moving average of the inflation rate of the past five years; it includes the current year and its lag does not. Age structure is the proportion of population older than 65. For detailed data definitions and sources, see Appendix B.

shown repeatedly in the literature that the fixed effects capture many country-specific unobservable factors that might affect both the dependent variable and the explanatory variables. Below we first explore the direct empirical relationship between long-term inflation and established entrepreneurship and then use an instrumental variable approach to identify the effects of exogenous changes of long-term inflation on established entrepreneurship.

3.2 Basic Regression Results

Table 2 reports the basic regression results. In either pooled ordinary least squares (OLS) or fixed effects OLS regressions, it appears that the effects of long-term inflation is weak or has the opposite sign, as predicted in theory. But these results are problematic because an econometric problem in the estimation of our empirical model (39) using OLS. The lagged dependent variable is included as a regressor because it is natural to expect some persistence in established entrepreneurship. But in both the fixed and random effects settings, the lagged dependent variable is correlated with the error term, even if we assume that the disturbances are not themselves autocorrelated. Fortunately, Arellano and Bond (1991) develop a consistent generalized method of moments (GMM) estimator that solves this problem. Column 3 of Table 2 reports

Table 2: Regression of Established Entrepreneurship in Labor Force (EELF)

	Pooled OLS (1)	Fixed effects OLS (2)	Arellano-Bond GMM (3)
	Dependent variable is EELF		
EELF_{t-1}	0.779***	0.250**	0.780***
	(0.0737)	(0.113)	(0.0675)
Inflation 5-yr MA_{t-1}	0.000939	-0.00106	0.00379**
	(0.000756)	(0.00193)	(0.00169)
Year fixed effects	Yes	Yes	Yes
Country fixed effects		Yes	Yes
AR(2) test			[0.085]
Hansen J test			[1.000]
Observations	465	465	363
Countries	74	74	62
R-square	0.695	0.196	

Notes: *, ** and *** are 10%, 5%, and 1% level of statistical significance, respectively. Pooled cross-sectional OLS regressions in column 1, with robust standard errors clustered by country in parentheses. Fixed effects OLS regressions in column 2, with country dummies and robust standard errors clustered by country in parentheses. GMM of Arellano-Bond in column 3 with robust standard errors; in this method we instrument for the dependent variable using a double lag, resulting in a smaller sample. Dependent variable is Established Entrepreneurship in Labor Force. Inflation 5-yr MA is the moving average of the inflation rate of the past five years; it includes the current year and its lag does not. Base sample is an unbalanced panel, 2002-2014, with yearly data. For detailed data definitions and sources, see Appendix B.

the results using their method by instrumenting for the dependent variable using a double lag. This reduces our sample size. Nevertheless, it shows that once we take care of the correlation between lagged dependent variable and the error term, there is a positive statistical relationship between long-term inflation and established entrepreneurship. Of course, our theory suggests a causal relationship between the two. These estimators do not necessarily identify the causal effects of long-term inflation on entrepreneurship. Below we also apply the instrumental variables approach to see if long-term inflation has any causal effect on entrepreneurship.

3.3 Instrumental Variable Estimates

For many reasons, inflation rates are endogenous, that is, many forces can shape the inflation experience of a country. Many of these forces might affect other macroeconomic variables, such as entrepreneurship, as well. This is why we cannot take the statistical relationship between inflation and other macro variables at face value. The estimation of causal effects requires exogenous sources of variation. Generally, it is hard to pick a plausible instrument for inflation in macroeconomics. While we do not have an ideal source of exogenous variation, there is a promising candidate: the age structure of a country. Specifically, we mean the proportion of the population that is over 65. Such choice is based on Bullard et al. (2012), who propose a political economy theory of how countries with an older population tend to choose lower inflation

Table 3: Regression of Long-term Inflation

	Pooled OLS	Fixed effects OLS	Arellano-Bond GMM
	(1)	(2)	(3)
	Dependent variable is inflation 5-yr MA_{t-1}		
Inflation 5-yr MA_{t-2}	0.901***	0.656***	0.606***
	(0.053)	(0.0565)	(0.0501)
Age $structure_{t-5}$	-0.0414***	-0.120*	-0.508***
	(0.0121)	(0.0649)	(0.0929)
Year fixed effects	Yes	Yes	Yes
Country fixed effects		Yes	Yes
AR(2) test			[0.250]
Hansen J test			[0.978]
Observations	411	411	411
Countries	74	74	74
R-squared	0.947	0.679	

Notes: *, ** and *** are 10%, 5%, and 1% level of statistical significance, respectively. Pooled cross-sectional OLS regressions in column 1, with robust standard errors clustered by country in parentheses. Fixed effects OLS regressions in column 2, with country dummies and robust standard errors clustered by country in parentheses. GMM of Arellano-Bond in column 3 with robust standard errors; in this method we instrument for the dependent variable using a double lag. Dependent variable is lagged inflation 5-yr MA. Inflation 5-yr MA is the moving average of the inflation rate of the past five years; it includes the current year and its lag does not. Age structure is the proportion of population older than 65. Base sample is an unbalanced panel, 2002-2014, with yearly data. For detailed data definitions and sources, see Appendix B.

rates. Relatedly, Burdett el al (2016) document that households above age 56 use relatively more cash than younger households. Thus, we expect the age structure to affect inflation rates.

Table 3 reports the results of age structure regressions on long-term inflation. Since our dependent variable is the moving average of inflation over the past five years, we choose the age structure five years ago as our explanatory variable, with the interpretation that the age structure of a country would affect the inflation rates for the next few years. Similar to Table 2, we again present results using pooled OLS, fixed effects OLS, and the Arellano-Bond GMM. There are mainly three things to note. First, the coefficient of age structure is significantly negative in all three methods. Second, the autocorrelation of the moving average of inflation is strong, as expected. This means we should rely on the Arellano-Bond GMM method to deal with the issue that the lagged dependent variable is correlated with the error term. Third, since the age structure can be seen as exogenous to monetary policy, we can interpret the relationship as causal. Column 3 shows that the effect of age structure on long-term inflation is strong, negative, and statistically significant.

But are there other channels through which age structure can affect entrepreneurship? First, the fraction of population that is over 65 is not mechanically related to our entrepreneurship measure; the measures from GEM are in terms of the percentage of the population that is below age 65. Second, one might expect that the age structure might be related to the entry of entrepreneurship. For example, young people may

Table 4: Regression of Established Entrepreneurship in Labor Force (EELF) with Age Structure$_{t-5}$ Instrument						
	(1)	(2)	(3)	(4)	(5)	(6)
	Arellano-Bond GMM; dependent variable is EELF					
EELF$_{t-1}$	0.327**	0.312**	0.309**	0.457***	0.410***	0.471***
	(0.137)	(0.145)	(0.134)	(0.155)	(0.154)	(0.134)
Inflation 5-yr MA$_{t-1}$	0.00695***	0.00726***	0.00732***	0.00500**	0.00801***	0.00622**
	(0.00222)	(0.00244)	(0.00201)	(0.00220)	(0.00310)	(0.00286)
Growth$_{t-1}$		-0.000153				0.00120
		(0.00115)				(0.000945)
Openness$_{t-1}$			-3.61e-06			7.12e-05
			(8.58e-05)			(0.000113)
Tertiary Enrollment$_{t-1}$				-6.14e-05		-3.05e-05
				(0.000195)		(0.000206)
Financial Depth$_{t-1}$					0.000163	0.000119
					(0.000116)	(9.48e-05)
Hansen J test	[0.999]	[1.000]	[0.998]	[1.000]	[0.999]	[1.000]
AR(2) test	[0.118]	[0.136]	[0.144]	[0.124]	[0.069]	[0.091]
Number of Observations	411	411	411	348	406	346
Number of Countries	74	74	74	64	73	63

Notes: *, ** and *** are 10%, 5%, and 1% level of statistical significance, respectively. GMM of Arellano-Bond is performed with robust standard errors; in this method we instrument for the dependent variable using the age structure five years ago. Inflation 5-yr MA is the moving average of the inflation rate of the past five years; it includes the current year and its lag does not. Age structure is the proportion of population older than 65. Base sample is an unbalanced panel, 2002-2014, with yearly data. Growth is the growth rate of GDP per capita, openness is the sum of exports and imports divided by GDP, and financial depth is the private debt/GDP ratio. For detailed data definitions and sources, see Appendix B.

be more likely to have innovative ideas that make entrepreneurship profitable. Or it may be easier for older people to overcome the liquidity constraint of entrepreneurship. These arguments, however, while having ambiguous overall effects, concerns the entry flow of entrepreneurship rather than the stock of entrepreneurs in the long run. While we do not have a precise theory for why age structure should have no direct effect on the stock of entrepreneurship, there is no particular reason that suggests otherwise. Furthermore, later we provide some useful empirical evidence suggesting that our age structure measure does not affect entrepreneurship in other channels.

Before that, however, we shall present our main results using age structure as an instrument, as shown in Table 4. In all specifications we use the Arellano-Bond GMM procedure and use age structure from five years ago as (the excluded) instrument. We do not use two-stage least squares because of the inclusion of the lagged dependent variable as the regressor.[13] We rely on the Hansen J test for the validity of our use of the instrumental variables. As shown, Table 4 suggests a statistically significant causal effect of inflation on entrepreneurship that is robust to the inclusion of many other control variables, such as growth, trade, education, and finance.

[13]If we were to perform two-stage least squares, we would need to include the lagged dependent variable in both stages. That poses another problem: then we should also include Inflation 5-yr MA$_{t-2}$ in the first stage, as shown in Table 3. But that would make the analysis even more complicated.

Table 5: Subsample Regression of Long-term Inflation

	Non-eurozone		Eurozone	
	Pooled OLS	Arellano Bond GMM	Pooled OLS	Arellano Bond GMM
	(1)	(2)	(3)	(4)
	Dependent variable is Inflation 5-yr MA$_{t-1}$			
Inflation 5-yr MA$_{t-2}$	0.902***	0.667***	0.901***	0.868***
	(0.0574)	(0.0596)	(0.0382)	(0.0247)
Age structure$_{t-5}$	-0.0437***	-0.511***	0.0280***	0.234
	(0.0134)	(0.109)	(0.00819)	(0.171)
Year fixed effects	Yes	Yes	Yes	Yes
Country fixed effects		Yes		Yes
AR(2) test		[0.255]		[0.172]
Hansen J test		[0.999]		[1.000]
Number of Countries	59	59	15	15
Number of Observations	289	289	122	122
R-square	0.944		0.922	

Notes: *, ** and *** are 10%, 5%, and 1% level of statistical significance, respectively. Pooled cross-sectional OLS regressions in columns 1 and 3, with robust standard errors clustered by country in parentheses. GMM of Arellano-Bond in columns 2 and 4, with robust standard errors; in this method we instrument for the dependent variable using a double lag. Dependent variable is lagged inflation 5-yr MA. Inflation 5-yr MA is the moving average of the inflation rate of the past five years; it includes the current year and its lag does not. Age structure is the proportion of population older than 65. Base sample is an unbalanced panel, 2002-2014, with yearly data. For detailed data definitions and sources, see Appendix B.

Validity of the Instrumental Variable

To further examine the validity of our instrumental variable, we use the following fact: the eurozone is a monetary union and shares the same monetary policy. Thus, the political economy theory of Bullard et al. (2012) should not work for the individual countries within the monetary union. Specifically, while we expect age structure to affect inflation in other countries, the age structure of individual countries is unlikely to affect the monetary policy of the whole monetary union. Table 5 provides evidence that is consistent with this hypothesis: the effect of age structure on long-term inflation is negatively significant in non-eurozone countries, as predicted by the theory of Bullard et al. (2012), and yet the effect disappears once we only look at the countries in the eurozone.

Once we have established the above differential effects of age structure on inflation, then we can ask whether the relationship between age structure and established entrepreneurship is the same in the two subsamples. If we consider that age structure could affect established entrepreneurship through some channels other than long-run inflation, then these other channels should be similar in the two subsamples. However, columns (1), (2), (4), and (5) in Table 6 suggest that we only have evidence that age structure is related to established entrepreneurship in non-eurozone countries, whereas the link in eurozone countries appears to be weak. This means we have no evidence that age structure affects established entrepreneurship through other chan-

Table 6: Subsample Regression of Established Entrepreneurship in Labor Force (EELF)					
	Non-eurozone			Eurozone	
	Pooled OLS	Arellano-Bond GMM	Arellano-Bond GMM	Pooled OLS	Arellano Bond GMM
	(1)	(2)	(3)	(4)	(5)
	Dependent variable is EELF				
$EELF_{t-1}$	0.821***	0.618***	0.542***	0.693***	0.660***
	(0.0735)	(0.115)	(0.124)	(0.0545)	(0.0613)
Age $structure_{t-5}$	-0.00117**	-0.00841***		-0.000911	0.00121
	(0.000512)	(0.00308)		(0.00117)	(0.00431)
5-yr average $inflation_{t-1}$			0.00523***		
			(0.00180)		
Year fixed effects	Yes	Yes	Yes	Yes	Yes
Country fixed effects		Yes	Yes		Yes
AR(2) test		[0.129]	[0.133]		[0.443]
Hansen J test		[1.000]	[1.000]		[1.000]
Number of Countries	59	49	59	15	14
Number of Observations	289	234	289	122	113
R-square	0.751			0.620	

Notes: *, ** and *** are 10%, 5%, and 1% level of statistical significance, respectively. Pooled cross-sectional OLS regressions in columns 1 and 3, with robust standard errors clustered by country in parentheses. GMM of Arellano-Bond in columns 2 and 4, with robust standard errors; in this method we instrument for the dependent variable using a double lag. Dependent variable is Established Entrepreneurship in Labor Force. Inflation 5-yr MA is the moving average of the inflation rate of the past five years; it includes the current year and its lag does not. Age structure is the proportion of population older than 65. Base sample is an unbalanced panel, 2002-2014, with yearly data. For detailed data definitions and sources, see Appendix B.

nels.[14] Of course, the size of the eurozone subsample is relatively small. But we can at least treat these results as suggestive that our choice of IV is valid. In column (3), we also re-estimate our IV regression with the non-eurozone sample, and can see that the effect of inflation is in line with our findings in Table 4.

4 Concluding Remarks

This paper studies how long-term inflation shapes entrepreneurial activities and shows that this channel can affect our understanding of monetary policy. We consider two timing assumptions and clarify how money flows between workers and entrepreneurs. Because we also allow agents to borrow, the usual criticism of cash-in-advance models does not apply here. It is clear then that the nominal interest rate contains the inflation tax, and whoever needs to use money for liquidity services has to pay the inflation tax. Of course, if we assume that no one needs money for liquidity, then no one has to pay the inflation tax and the classical dichotomy applies. In this case, neither inflation nor nominal interest matters.

The theory in this paper predicts that long-run inflation encourages entrepreneur-

[14]We get similar results if we use age $structure_{t-1}$ instead of age $structure_{t-5}$.

ship, and is supported by our empirical results using an instrumental variables approach. The effect of monetary policy on output in HBC countries, can therefore be qualitatively different than that in LBC countries. However, similar to Lagos and Rocheteau (2005), this theory offers another subtle lesson. Even though steady-state inflation can raise output in some circumstances, it does not increase welfare. In the context of this paper, steady-state inflation always reduces welfare even though it can increase output. But here is an even more subtle caveat: there could be positive externalities of entrepreneurial activities to growth that are not captured in this paper (see the models in Silviera and Wright 2010 and Chiu et al. 2017). Because these externalities might have important long-term implications, we should be cautious in judging the welfare consequences of long-run inflation.

For future extensions, one can incorporate heterogeneity, such as entrepreneurial ability, into the model. Note that in this paper, there are no wealth effects because of the quasi-linear utility structure so as to make the model as simple as possible. Another extension, therefore, would be to abandon this structure so as to study quantitatively how inflation affects entrepreneurship and the wealth distribution. But these considerations should not change the underlying mechanisms of this paper. Adding endogenous growth would also be another exciting direction to take.

Appendix A: Proof of Proposition 1

Note that $f_\ell(\frac{1-n}{n}) = \frac{\alpha n f(\frac{1-n}{n})}{1-n}$,$u(x) = \frac{x^{1-\rho}-1}{1-\rho}$, $f(\ell) = \ell^\alpha$. Then equation (18) can be reduced to

$$[n^{1-\alpha}(1-n)^\alpha]^{1-\rho}[\frac{1-\alpha}{n} - \frac{\alpha\beta}{1+\pi}\frac{1}{1-n}] + (v_w - v_e) = 0.$$

(1) Given $\rho = 1$, we then have

$$[\frac{1-\alpha}{n} - \frac{\alpha\beta}{1+\pi}\frac{1}{1-n}] + (v_w - v_e) = 0,$$

which is the case of log utility discussed in the paper. We know that an increase in π would increase n.

(2) Given $\rho \neq 1$, we then have

$$[n^{1-\alpha}(1-n)^\alpha]^{1-\rho}[\frac{1-\alpha}{n} - \frac{\alpha\beta}{1+\pi}\frac{1}{1-n}] = v_e - v_w.$$

Let $g\,(n,\pi) = [n^{1-\alpha}(1-n)^\alpha]^{1-\rho}[\frac{1-\alpha}{n} - \frac{\alpha\beta}{1+\pi}\frac{1}{1-n}]$. The total differential would then be

$$\frac{\partial g\,(n,\pi)}{\partial n}dn + \frac{\partial g\,(n,\pi)}{\partial \pi}d\pi = 0. \tag{40}$$

Note that

$$\begin{aligned}
\frac{\partial g\,(n,\pi)}{\partial n} &= -\frac{\partial}{\partial n}[(1-\alpha)\,n^{(1-\alpha)(1-\rho)-1}\,(1-n)^{\alpha(1-\rho)} - \frac{\alpha\beta}{1+\pi}n^{(1-\alpha)(1-\rho)}\,(1-n)^{\alpha(1-\rho)-1}] \\
&= -n^{(1-\alpha)(1-\rho)-2}\,(1-n)^{\alpha(1-\rho)-2}\,[\left(\frac{\alpha\beta}{1+\pi} + 1 - \alpha\right)\rho n^2 \\
&\quad + (1-\alpha)\left(\alpha\left(1 + \frac{\beta}{1+\pi}\right) + \left(2 - \alpha - \frac{\alpha\beta}{1+\pi}\right)\rho\right)n + (1-\alpha)\,(\alpha + (1-\alpha)\,\rho)].
\end{aligned}$$

Given $\alpha, \beta, n \in (0,1)$, $\rho > 0$, $Z > 0$, then $2 - \alpha - \frac{\alpha\beta}{1+\pi} > 0$ and $\partial g\,(n,\pi)\,/\partial n < 0$. Also note that

$$\frac{\partial g\,(n,\pi)}{\partial \pi} = [n^{1-\alpha}(1-n)^\alpha]^{1-\rho}[\frac{\alpha\beta}{(1+\pi)^2}\frac{1}{1-n}] > 0.$$

Then from (40) we have $dn/d\pi > 0$. Q.E.D.

Appendix B: Variable Definitions and Sources

Established Entrepreneurship. Percentage of age 18-64 population who are currently an owner-manager of an established business (i.e., owning and managing a running business that has paid salaries, wages, or any other payments to the owners for more than 42 months). Source: Global Entrepreneurship Monitor (http://www.gemconsortium.org/data).

Inflation Rate. Inflation as measured by the consumer price index reflects the annual percentage change in the cost to the average consumer of acquiring a basket of goods and services that may be fixed or changed at specified intervals, such as yearly (annual %). Source: World Development Indicators. Coverage: 2004-2014.

Average Inflation of Past Five Years. Here we use the past five-year CPI index geometric average. Source: World Development Indicators.

Growth Rate of GDP Per Capita . The growth rate of real GDP per capita (%). Source: World Development Indicators Coverage: 2004-2014. Minimum and maximum number of countries in any year are 163 and 181, respectively.

Openness. Share of merchandise exports and imports at current PPPs. Source: World Development Indicators.

Table B1 : Descriptive Statistics for Table 5 and 6				
	Mean	*Std. Dev.*	*Obs*	*Countries*
Panel E				
Non-euro Subsample				
Inflation 5-yr MA_{t-1}	5.123	3.987	289	59
Age structure$_{t-5}$	10.409	4.812	289	59
EELF	.113	.072	289	59
Eurozone Subsample				
Inflation 5-yr MA_{t-1}	2.683	1.498	122	15
Age structure$_{t-5}$	16.268	2.190	122	15
EELF	.091	.042	122	15
Panel F				
Non-euro Subsample				
EELF	.105	.062	234	49
Age structure$_{t-5}$	11.288	4.638	234	49
Eurozone Subsample				
EELF	.092	.043	113	14
Age structure$_{t-5}$	16.266	2.160	113	14

Notes: Panel E is for Table 5, and columns 1, 3 and 4 in Table 6. Panel F is for column 2 and 5 in Table 6. Inflation 5-yr MA is the moving average of the inflation rate of the past five years; it includes the current year and its lag does not. Age structure is the proportion of population older than 65. For detailed data definitions and sources, see Appendix B.

Tertiary Enrollment: Total enrollment in tertiary education (ISCED 5 to 8), regardless of age, expressed as a percentage of the total population of the five-year age group following on from secondary school leaving. Source: World Development Indicators.

Age structure. Population age 65 and above as a percentage of the total population. Population is based on the de facto definition of population, which counts all residents regardless of legal status or citizenship. Source: World Bank staff estimates based on age distributions of United Nations Population Division's World Population Prospects.

Ease of Doing Business index: Ease of doing business ranks economies from 1 to 190, with first place being the best. A high ranking (a low numerical rank) means that the regulatory environment is conducive to business operation. The index averages the country's percentile rankings on 10 topics covered in the World Bank's Doing Business. The ranking on each topic is the simple average of the percentile rankings on its component indicators. Source: World Bank, Doing Business project.

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